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Fundamental Constants and the International System of Units

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Outline

- Task Group on Fundamental Constants
- Recommended values of the constants
- International System of Units (SI)
- Problems with the SI
- Definition of the meter
- Possible new kilogram
- Possible new ampere, kelvin, and mole
- Effect on the fundamental constants
- Effect on the task group
- Timing of the changes
- Conclusion

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CODATA recommended values of the fundamental physical constants: 2002

Peter J. Mohr, Barry N. Taylor

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2002 CODATA RECOMMENDED VALUES OF THE FUNDAMENTAL CONSTANTS OF PHYSICS AND CHEMISTRY NIST SP 959 (Apr/2005)

Values from: P. J. Mohr and B. N. Taylor, *Rev. Mod. Phys.* **77**, 1 (2005). The number in parenthesis is the one-sigma ($1\,\sigma$) uncertainty in the last two digits of the given value.

Quantity	Symbol	Numerical value	Unit
speed of light in vacuum	c, c_0	299 792 458 (exact)	m s^{-1}
magnetic constant	μ_0	$4\pi \times 10^{-7}$ (exact)	N A^{-2}
electric constant $1/(\mu_0 c^2)$	ϵ_0	$8.854\,187\,817\ldots \times 10^{-12}$	F m^{-1}
Newtonian constant of gravitation	G	$6.6742(10) \times 10^{-11}$	m^3 kg^{-1} s^{-2}
Planck constant	h	$6.626\,0693(11) \times 10^{-34}$	J s
$h/(2\pi)$	$\hbar$	$1.054\,571\,68(18) \times 10^{-34}$	J s
elementary charge	e	$1.602\,176\,53(14) \times 10^{-19}$	C
fine-structure constant $e^2/(4\pi\epsilon_0\hbar c)$	α	$7.297\,352\,568(24) \times 10^{-3}$	
inverse fine-structure constant	α^{-1}	137.035 999 11(46)	
Rydberg constant $\alpha^2 m_e c/(2h)$	R_∞	10 973 731.568 525(73)	m^{-1}
Bohr radius $\alpha/(4\pi R_\infty)$	a_0	$0.529\,177\,2108(18) \times 10^{-10}$	m
Bohr magneton $e\hbar/(2m_e)$	μ_B	$927.400\,949(80) \times 10^{-26}$	J T^{-1}

Quantity	Symbol	Numerical value	Unit
electron mass	m_e	$9.109\,3826(16) \times 10^{-31}$	kg
proton mass	m_p	$1.672\,621\,71(29) \times 10^{-27}$	kg
proton-electron mass ratio	m_p/m_e	1836.152 672 61(85)	
Avogadro constant	N_A, L	$6.022\,1415(10) \times 10^{23}$	mol^{-1}
Faraday constant $N_A e$	F	96 485.3383(83)	C mol^{-1}
molar gas constant	R	8.314 472(15)	J mol^{-1} K^{-1}
Boltzmann constant R/N_A	k	$1.380\,6505(24) \times 10^{-23}$	J K^{-1}
Stefan-Boltzmann const. $\pi^2 k^4/(60\hbar^3 c^2)$	σ	$5.670\,400(40) \times 10^{-8}$	W m^{-2} K^{-4}
magnetic flux quantum $h/(2e)$	Φ_0	$2.067\,833\,72(18) \times 10^{-15}$	Wb
Josephson constant $2e/h$	K_J	$483\,597.879(41) \times 10^9$	Hz V^{-1}
von Klitzing constant h/e^2	R_K	25 812.807 449(86)	Ω
electron volt (e/C) J	eV	$1.602\,176\,53(14) \times 10^{-19}$	J
(unified) atomic mass unit $\frac{1}{12}m(^{12}\mathrm{C})$	u	$1.660\,538\,86(28) \times 10^{-27}$	kg

A more extensive listing of constants is available in the reference given above and on the NIST Physics Laboratory Web site physics.nist.gov/constants.

NIST **National Institute of Standards and Technology**
Technology Administration, U.S. Department of Commerce

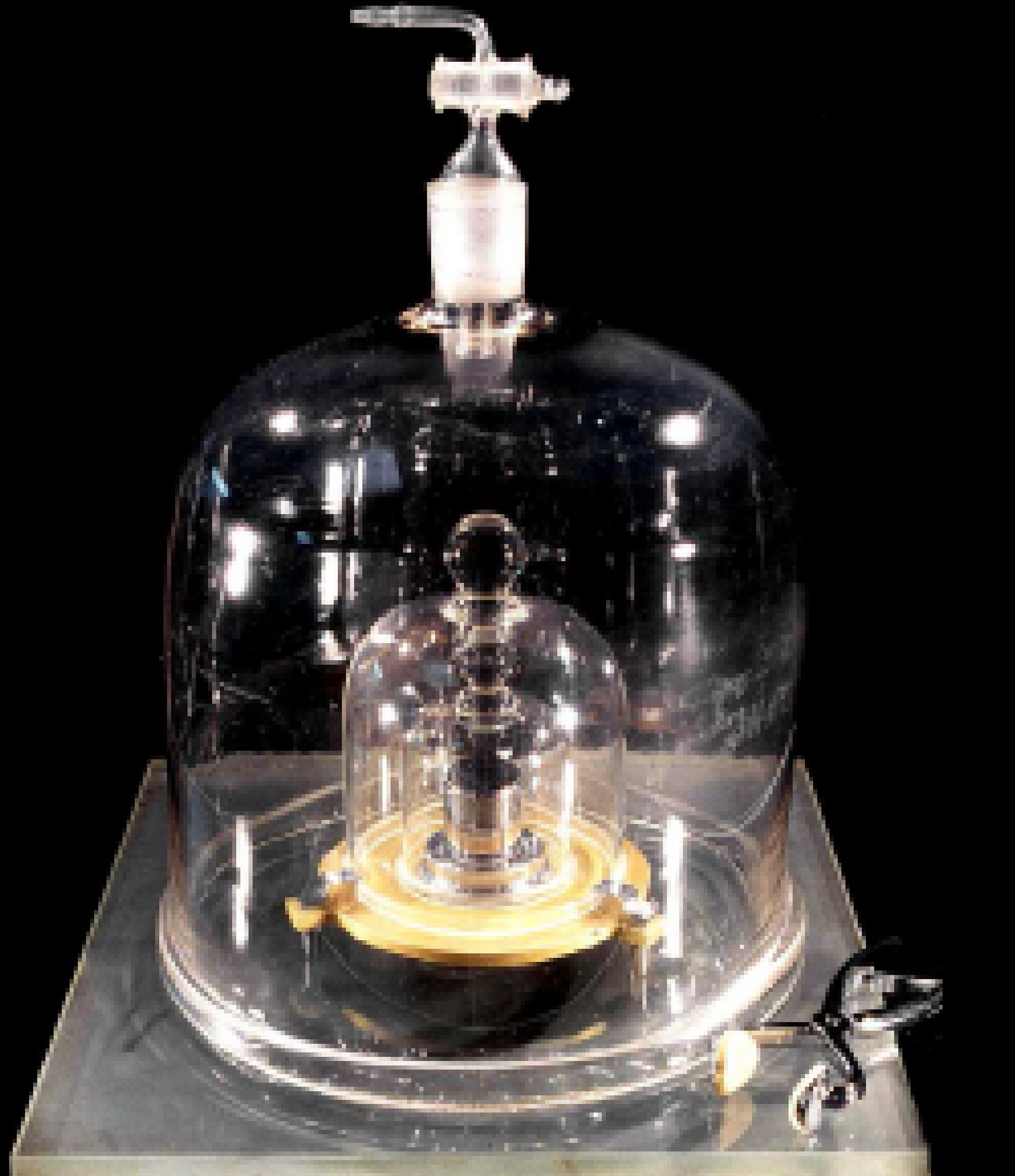
CODATA Fundamental Constant wallet cards

Some of the Fundamental Constants of Physics

- Newtonian constant of gravitation:
  $\boldsymbol{G = 6.6742(10) \times 10^{-11}\ \mathrm{m^3\ kg^{-1}\ s^{-2}}}$ $[1.5 \times 10^{-4}]$
- Avogadro constant:
  $\boldsymbol{N_{\mathrm{A}} = 6.022\,1415(10) \times 10^{23}\ \mathrm{mol^{-1}}}$ $[1.7 \times 10^{-7}]$
- electron mass:
  $\boldsymbol{m_{\mathrm{e}} = 9.109\,3826(16) \times 10^{-31}\ \mathrm{kg}}$ $[1.7 \times 10^{-7}]$
- Planck constant:
  $\boldsymbol{h = 6.626\,0693(11) \times 10^{-34}\ \mathrm{J\ s}}$ $[1.7 \times 10^{-7}]$
- fine-structure constant:
  $\boldsymbol{\alpha = 1/137.035\,999\,11(46)}$ $[3.3 \times 10^{-9}]$
- electron mass (in u):
  $\boldsymbol{m_{\mathrm{e}} = 5.485\,799\,0945(24) \times 10^{-4}\ \mathrm{u}}$ $[4.4 \times 10^{-10}]$
- Rydberg constant:
  $\boldsymbol{R_{\infty} = 10\,973\,731.568\,525(73)\ \mathrm{m^{-1}}}$ $[6.6 \times 10^{-12}]$

International System of Units (SI)

- **SI base units and symbols**
  - meter m (length)
  - kilogram kg (mass)
  - second s (time)
  - ampere A (electric current)
  - kelvin K (thermodynamic temperature)
  - mole mol (amount of substance)
  - candela cd (luminous intensity)
- **Some SI derived units and symbols**
  - hertz Hz (frequency)
  - newton N (force)
  - joule J (energy)
  - coulomb C (electric charge)
  - volt V (electric potential difference)
- **Non-SI units and symbols**
  - electron volt eV (energy)
  - unified atomic mass unit u (mass)



Limitations of the current kilogram prototype definition

- The prototype definition is not linked to an unchanging property of nature.
- The mass of the international prototype appears to be changing relative to the mass of its copies.
- The drift of the kilogram prototype together with its copies (relative to an unchanging standard) could be as large as 20×10^{-9} kg per year (Davis 2003).
- The prototype and its copies appear to gain mass over time and lose mass when washed for use in comparisons.
- The kilogram mass definition cannot be realized independently of the international prototype.

Possible redefinitions of the kilogram

- The limitations on stability of the definition of the kilogram in terms of the international prototype could be eliminated if the kilogram were defined in terms of a fundamental constant in analogy with the definition of the meter.
- 17th CGPM in 1983:
  - The meter is the length of the path traveled by light in vacuum during a time interval of 1/299 792 458 of a second.
- As a consequence for the velocity of light *c*:

$$c = \frac{1\ \mathrm{m}}{1/299\,792\,458\ \mathrm{s}} = 299\,792\,458\ \mathrm{m/s}$$

- An alternative statement of the definition could be:
  - The meter is the length scaled such that the velocity of light is 299 792 458 m/s.

Experimental realization of the kilogram (watt-balance experiment)

Kibble, Robinson, and Belliss (1990)
Williams Steiner, Newell, and Olsen (1998)
Steiner, Williams, Newell and Liu (2005)

- **Current interpretation:**
  - precise kilogram mass + watt-balance experiment → h
- **Alternative interpretation:**
  - precise kilogram mass ← watt-balance experiment + h
- **This suggests a possible new definition of the kilogram:**
  - The kilogram is the unit of mass scaled such that the Planck constant is exactly 6.626 069 3 x 10^{-34} J s.

Experimental realization of the kilogram (X-ray-crystal-density method)

Deslattes, Henins, Bowman, Schoonover, Carroll, Barnes, Machlan, Moore, Shields (1974)

International Avogadro Project (2004-2010)

Current interpretation:

- known mass silicon sphere + volume measurement + lattice spacing measurement + silicon isotopic composition measurement → N_A

- Alternative interpretation:
  - known mass silicon sphere ← volume measurement + lattice spacing measurement + silicon isotopic composition measurement + N_A
- This suggests a possible new definition of the kilogram:
  - The kilogram is the unit of mass scaled such that the Avogadro constant is exactly 6.022 141 527 x 10^{23} mol^{-1}.

Relation between the Avogadro constant N_A and the Planck constant h

- **Rydberg constant definition:**

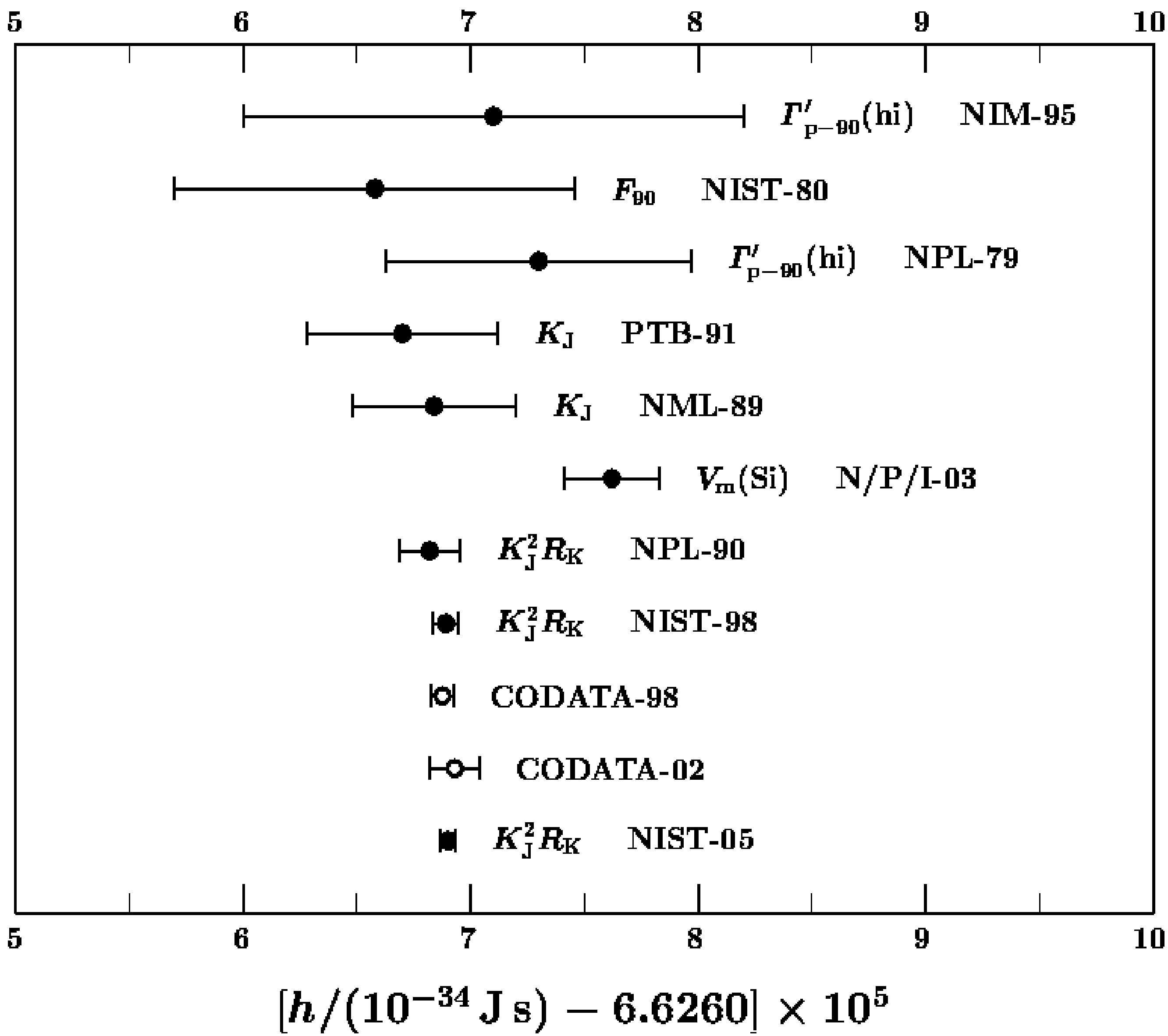
$$R_\infty = \frac{\alpha^2 m_e c}{2h} \quad \Rightarrow \quad \frac{1}{m_e} = \frac{\alpha^2 c}{2hR_\infty}$$

- **unified atomic mass unit u:**

$$m_e = A_r(e)\,u \quad \Rightarrow \quad \frac{1}{u} = \frac{A_r(e)}{m_e}$$

- **Avagadro constant:**

$$N_A = \frac{10^{-3}\ \mathrm{kg/mol}}{1\ \mathrm{u}} = A_r(e)\left(\frac{\alpha^2 c}{2hR_\infty}\right)\ 10^{-3}\ \mathrm{kg/mol}$$



5
6
7
8
9
10
$\Gamma'_{p-90}(\mathrm{hi})$ NIM-95
F_{90} NIST-80
$\Gamma'_{p-90}(\mathrm{hi})$ NPL-79
K_J PTB-91
K_J NML-89
$V_m(\mathrm{Si})$ N/P/I-03
$K_J^2 R_K$ NPL-90
$K_J^2 R_K$ NIST-98
CODATA-98
CODATA-02
$K_J^2 R_K$ NIST-05
$[h/(10^{-34}\ \mathrm{J\,s}) - 6.6260] \times 10^5$

Possible ampere redefinition

to make e exact

The ampere is the electric current scaled such that the elementary charge is 1.602 176 53 x 10^{-19} coulomb.

Possible kelvin redefinition

to make k exact

The kelvin, the unit of thermodynamic temperature, is scaled such that the Boltzmann constant is exactly 1.380 650 5 x 10^{-23} joule per kelvin.

Possible mole redefinition

to make N_A exact

The mole, the unit of amount of substance, is scaled such that the Avogadro constant is exactly 6.022 141 5 x 10^{23} per mole.

Effect of new SI on the uncertainties of some fundamental constants

G:	$1.5 \times 10^{-4} \rightarrow 1.5 \times 10^{-4} + \varepsilon$
N_A:	$1.7 \times 10^{-7} \rightarrow$ exact
m_e:	$1.7 \times 10^{-7} \rightarrow 6.6 \times 10^{-9}$
h:	$1.7 \times 10^{-7} \rightarrow$ exact
α:	$3.3 \times 10^{-9} \rightarrow 3.3 \times 10^{-9}$
m_e (amu):	$4.4 \times 10^{-10} \rightarrow 4.4 \times 10^{-10}$
R_∞:	$6.6 \times 10^{-12} \rightarrow 6.6 \times 10^{-12}$

$m(\mathcal{K})$: exact $\rightarrow$ parts in 10^8

Some of the other quantities that become exact in the new SI

Faraday constant
molar gas constant
Stefan-Boltzmann constant
Josephson constant
von Klitzing constant
eV-joule conversion factor
Hz-joule conversion factor
kelvin-joule conversion factor

Effect on the task group work

- The work of the task group on fundamental constants, although changed in some ways, would continue after the redefinitions.
- Many constants, such as the fine-structure constant and Rydberg constant, would still be determined by experiment and theory, and it will be necessary to recommend values.
- The mass of the prototype kilogram, which would still be used for practical international comparisons, would have a CODATA recommended value.
- Comparisons of the values of constants determined by various methods, as is done in the analysis leading to recommended values, would still provide tests the consistency of precision measurements and fundamental theory.

Timing of the changes: contributing publications

Redefinition of the kilogram: a decision whose time has come, Metrologia 42, 71 (2005).

and

Redefinition of the kilogram, ampere, kelvin and mole: a proposed approach to implementing CIPM recommendation 1 (CI-2005), Metrologia 43, 227 (2006).

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Recommendation to the Consultative Committee on Units
from
The CODATA Task Group on Fundamental Constants
on the
Redefinition of SI Base Units
(28 June 2005)

The CODATA Task Group

considering

- the importance of the fundamental constants to the scientific community because of their role in relating different branches of pure and applied science,
- the importance of defining the base units of the SI in terms of invariant physical quantities, and
- the significant reduction in uncertainty associated with the values of many fundamental constants that would occur if the Planck constant, the elementary charge, and the Boltzmann constant were to have exact values,

recommends to the CCU

- that the kilogram be redefined in terms of a fixed value of the Planck constant,
- that the ampere be redefined in terms of a fixed value of the elementary charge,
- that the kelvin be redefined in terms of a fixed value of the Boltzmann constant,
- that these changes be implemented at the same time as soon as conveniently possible, and
- that the values chosen for these constants be the CODATA recommended values at the time,

and also recommends that consideration be given to redefining at the same time the mole in terms of a fixed value of the Avogadro constant.

Time scale for possible redefinitions

- At its meeting in October 2004, the International Committee on Weights and Measures (CIPM) asked the Consultative Committee on Units (CCU) to study the possiblity of a fundamental constant-based definition of the kilogram.

- At its meeting in June 2005, the CCU requested that the CIPM approve preparation possible new definitions of the kilogram, ammpere, and kelvin in terms of fundamental constants and also consider redefinining the mole at the same time.

- At its meeting in October 2005, the CIPM approved, in principle, preparation of the new definitions, as requested by the CCU, for possible adoption by the General Conference on Weights and Measures (CGPM) in 2011, provided the results of experiments over the next few years are acceptable.

Conclusion

- The SI can be improved by modernizing the way units are defined.
- In particular, the definitions of the kilogram, ampere, kelvin and mole are based on 19th century science and technology and can be replaced by ones that take into account subsequent progress in physics.
- If an update of the SI is done by specifying values of the fundamental constants as discussed, the concepts of base units and derived units would not be not necessary.
- If there are no persistent problems with experiments, the changes could be made in 2011.
- Beyond the changes, CODATA recommended values of the fundamental constants would continue to be of value to science, technology, and commerce.