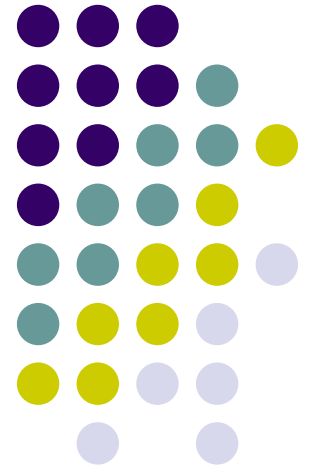


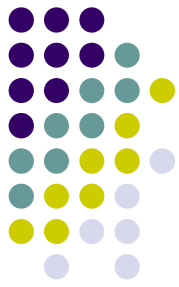
# Dissemination of 3D Visualization of Complex Function Data For the NIST Digital Library of Mathematical Functions

Qiming Wang, Sandy Ressler,  
Bonita Saunders

National Institute of Standards and Technology,  
USA

[qwang@nist.gov](mailto:qwang@nist.gov), [sressler@nist.gov](mailto:sressler@nist.gov),  
[bonita.saunders@nist.gov](mailto:bonita.saunders@nist.gov)

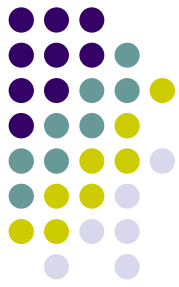




# Outline

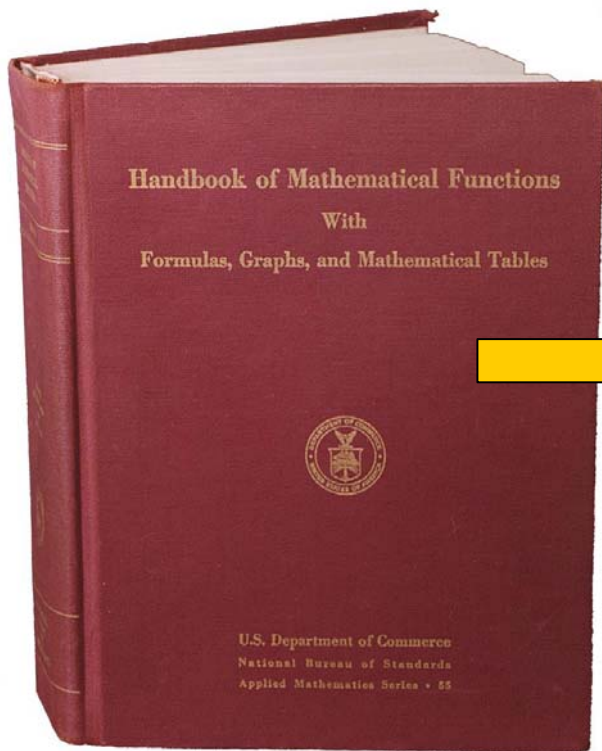
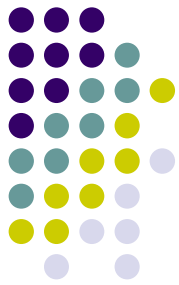
- NIST Digital Library of Mathematical Functions (DLMF) Project
- 3D visualization of complex mathematical functions
  - Data generation for special features
  - Web based Interactive visualization
- Dissemination of 3D visualization
  - VRML/X3D
  - Embed interactive 3D for PDFdocumentation
  - Animation of VRML DLMF
  - Quicktime VR
- Conclusion and future work

# NIST DLMF Project *(<http://dlmf.nist.gov>)*



- Update the “Handbook of Mathematical Functions,” Abramowitz and Stegun, eds., 1964.
- Publish the handbook on the Web as a Digital Library of Mathematical Functions, including formulas, references, computational methods, graphics.
- Incorporate state-of-the-art features for navigation, searching and 3D visualization.

# NIST Digital Library of Mathematical Functions



NIST Digital Library of Mathematical Functions - Microsoft Internet Explorer

Address: http://dlmf.nist.gov

Digital Library of Mathematical Functions

Ch. AI Airy and Related Functions by Frank W. J. Olver

NIST National Institute of Standards and Technology

Chapter AI: Airy and Related Functions [Notes \(draft\)](#)

by Frank W. J. Olver

AI.2(i) Airy's Equation [Notes](#)

AI.2.1  $\frac{d^2 w}{dz^2} = zw.$

Standard solutions are

AI.2.2  $w = \text{Ai}(z), \text{Bi}(z), \text{Ai}(ze^{-2\pi i/3}), \text{Ai}(ze^{2\pi i/3}).$

All solutions are entire functions of  $z.$

AI.2(ii) Initial Values [Notes](#)

AI.2.3  $\text{Ai}(0) = \frac{1}{3^{2/3}\Gamma(\frac{2}{3})} = 0.258819$

AI.2.4  $\text{Ai}'(0) = -\frac{1}{3^{1/3}\Gamma(\frac{1}{3})} = -0.498010$

AI.2.5  $\text{Bi}(0) = \frac{1}{3^{1/6}\Gamma(\frac{2}{3})} = 0.498010$

AI.2.6  $\text{Bi}'(0) = \frac{3^{1/6}}{\Gamma(\frac{1}{3})} = 0.448288$

AI.2(iii) Numerically Satisfactory Div...

Figure AL3.6:  $\Im \text{Ai}(z + iy)$



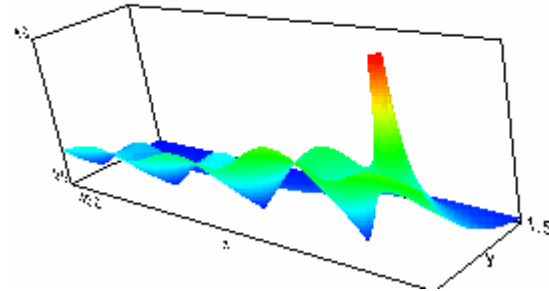
# Reasons to Update Handbook

- Continued interest in handbook even though outdated
- Advances in mathematical and computational techniques associated with classical special functions
- Identification of new functions having widespread importance in emerging applications

# Why need 3D visualization in DLMF?



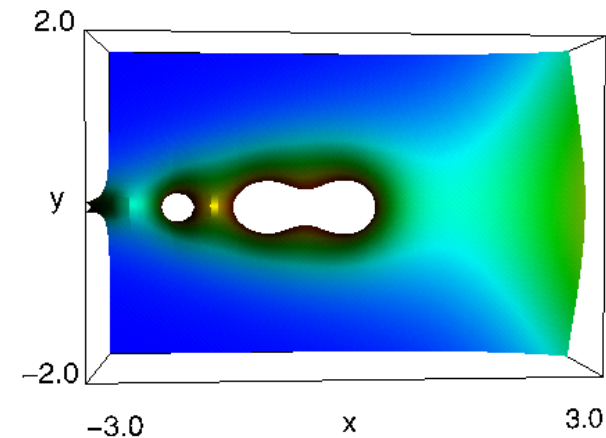
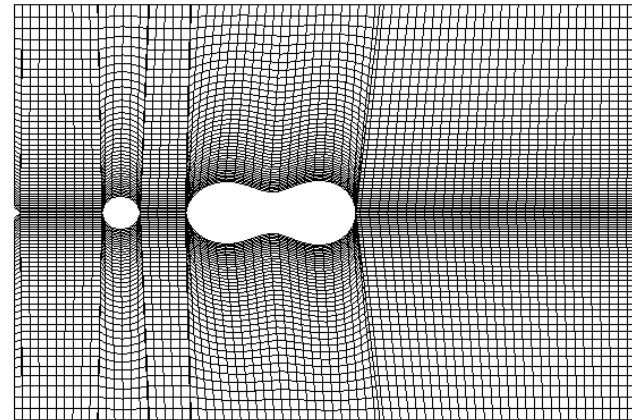
- Complexities of special mathematical functions
- Technology exists for interactive graphics on Web
- Good graphics aids researchers
- Material accessible to wider audience
- Stimulates interest and additional research



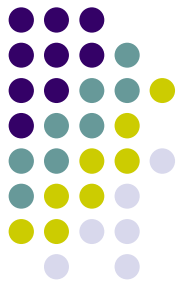
# Procedure to generate data for 3D visualization



- Generate grid to capture significant features of the function
- Contour fitted grid generation to solve the clipping problems using some software package
- Function values on the grid are calculated from Mathematica, Maple, or special codes



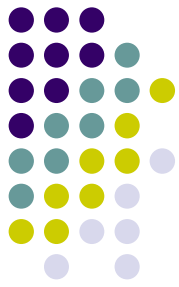
# Some important issues of 3D visualization



- Web based visualization
- User interaction during the visualization
- Portable file format



# Advantages of Using VRML/X3D in the DLMF



VRML /X3D

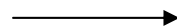


- Web accessible



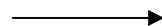
- Browsers exist for a variety of platforms and can be downloaded free of charge on web

- Portable file formats



- VRML/X3D is a standard 3D file format for Web based visualization. Many applications have the functions to import or export VRML/X3D

- User interaction

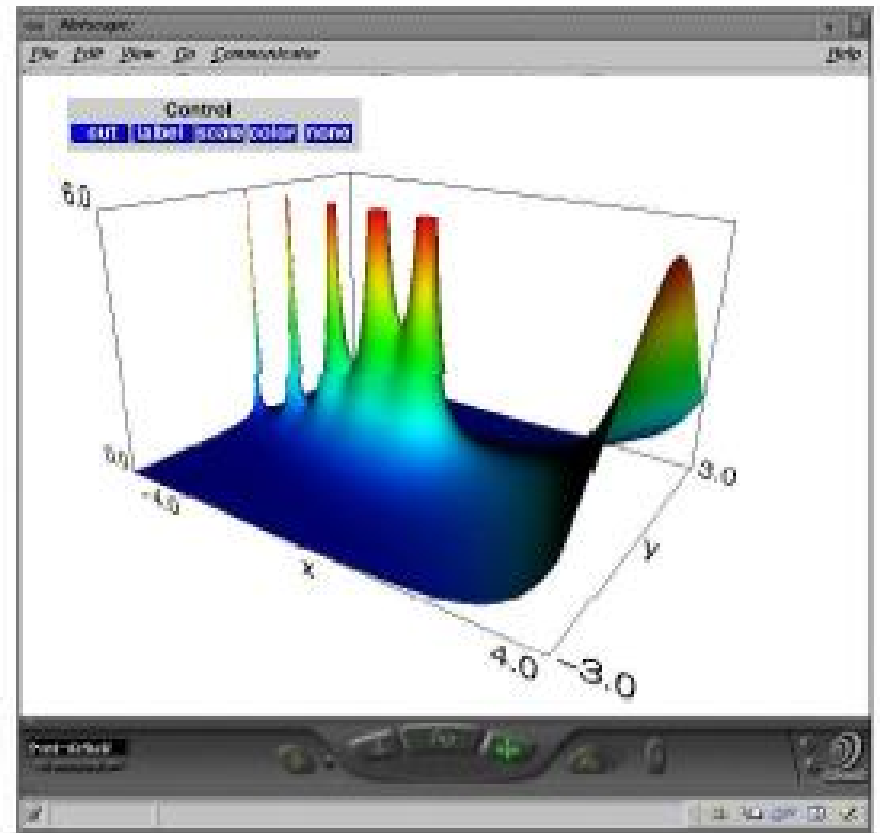


- Script nodes in addition to Browse navigation functions

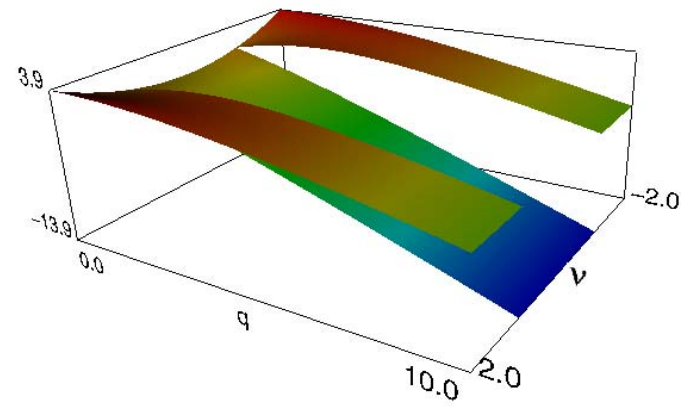
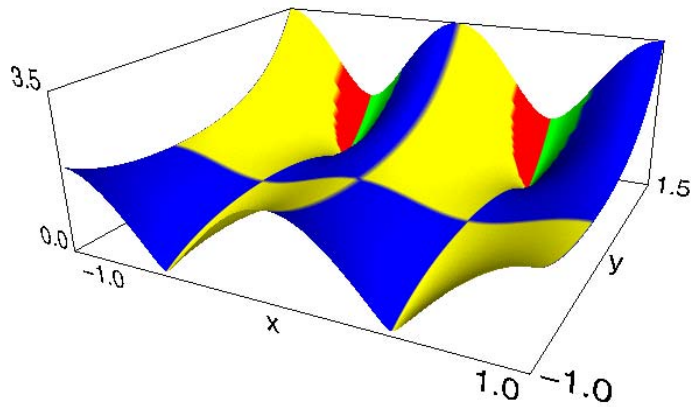
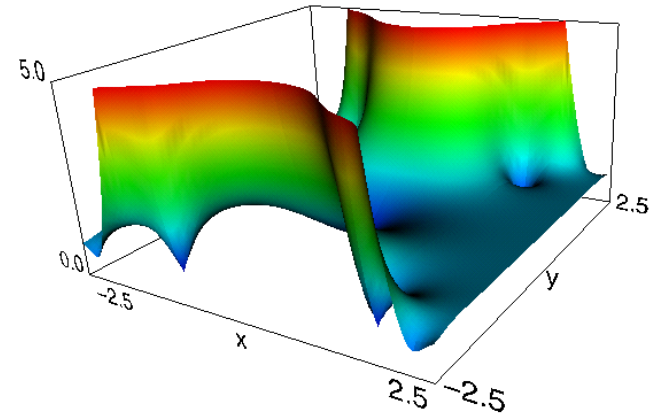
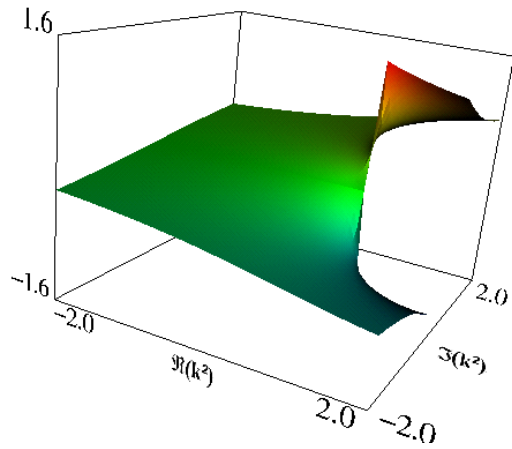
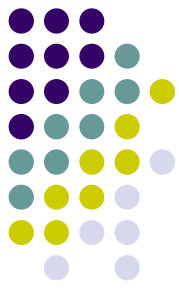


# MathViewer

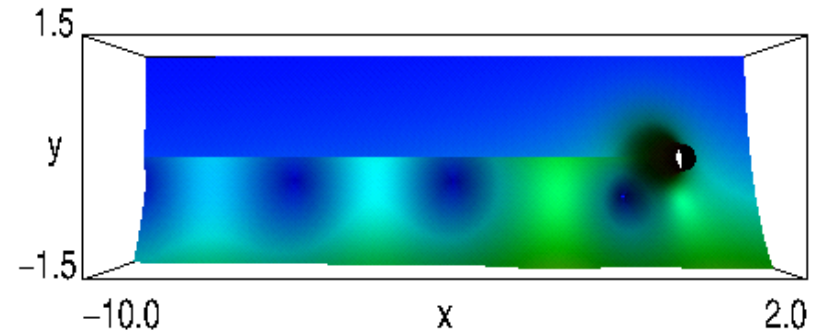
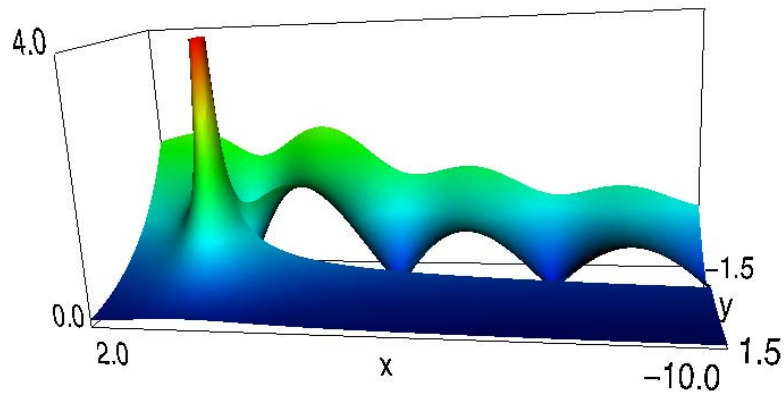
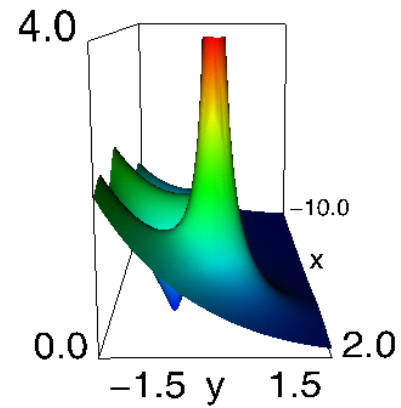
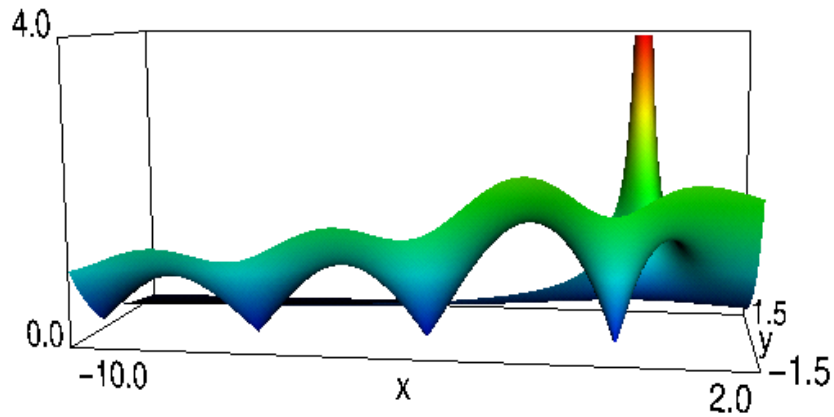
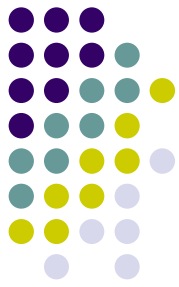
MathViewer is a VRML prototype node designed to implement interactive visualization of mathematical functions.



# Examples



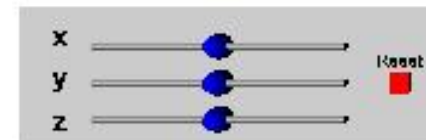
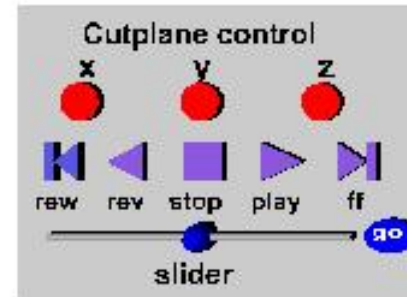
# VRML Display of Hankel Function from Different Viewpoints



# Interaction Features of MathViewer



- Dynamic cutting plane control
- Axis and labeling style control
- Color mapping control
- Scaling control





# Adobe Acrobat 3D

- Publish, share, review, and mark up 3D designs in Intelligent Documents.
- Easily insert, publish, and edit 3D designs
  - Insert 3D designs into existing and new Microsoft Office documents using the OLE Control Extensions(OCX) included in Acrobat 3D.
  - Use the Acrobat 3D toolkit to add material, create animations such as exploded views, edit lighting, and save as a 3D object or 2D raster/vector image.
  - Add JavaScript to any Adobe PDF file that contains 3D designs to convey deeper levels of information.

# Embed 3D DLMF to PDF Document

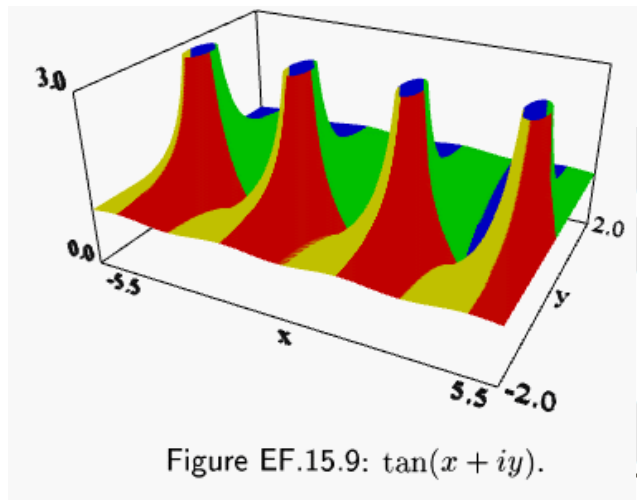


- One production of the DLMF is a hard copy of the handbook. Each chapter has one PDF file with 3D figures shown as 2D images.
- Using Acrobat 3D, we can hide a 3D figure at the location of 2D image. When user clicks the 2D image, 3D figure will display with a tool bar. Then user can interactively browse the complex figure.

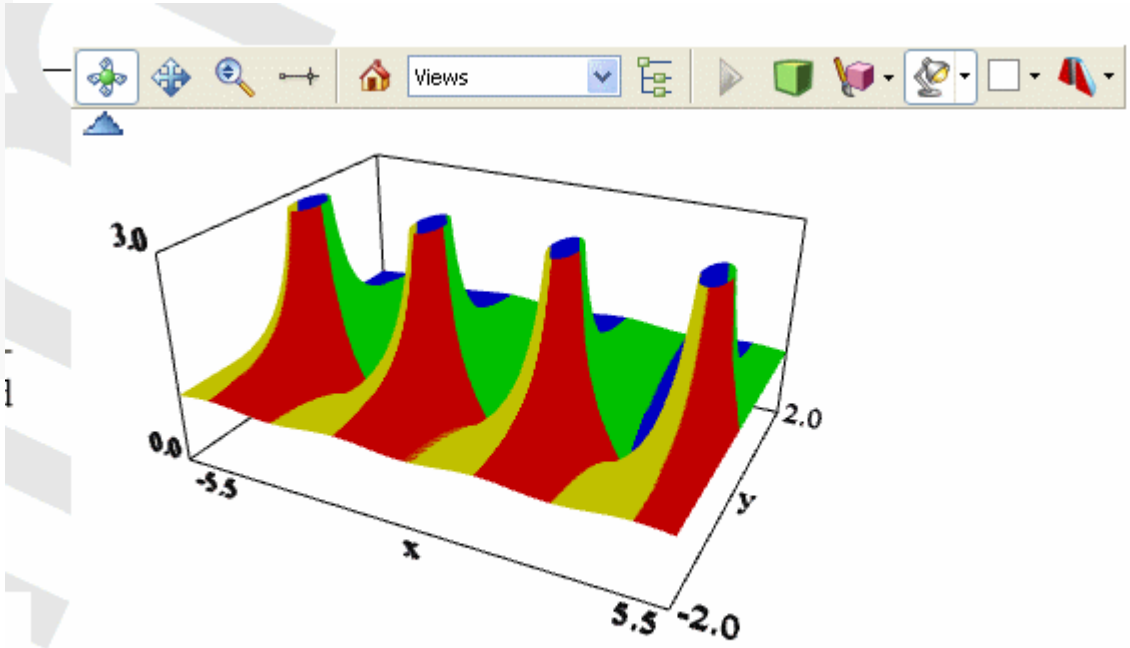


# Example

2D image

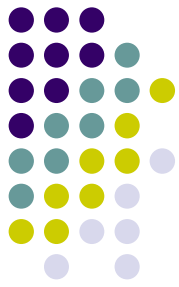


3D figure

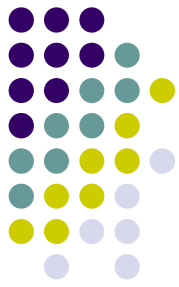




# Main Navigation Functions of embedded 3D figure

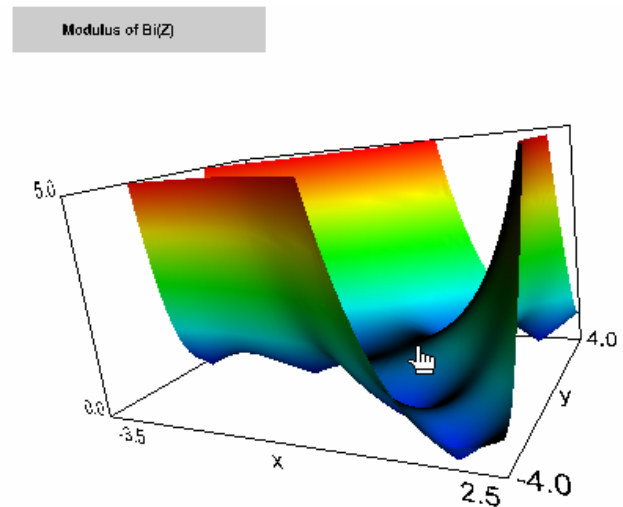


- Rotate, Move, Zoom
- Viewpoint management
- Background
- [Example chapter - EF](#)
- [Example chapter – GA](#)

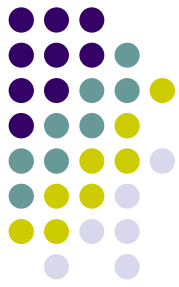


# Animation version

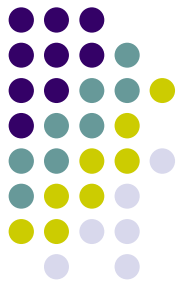
- Animation for users without VRML access
- The procedure of generating animation:
  - Create an alternate version of VRML file for generating animation movie
  - Browse in a VRML Browser
  - Capture the movie using video capture, or other tools



# QuickTime VR

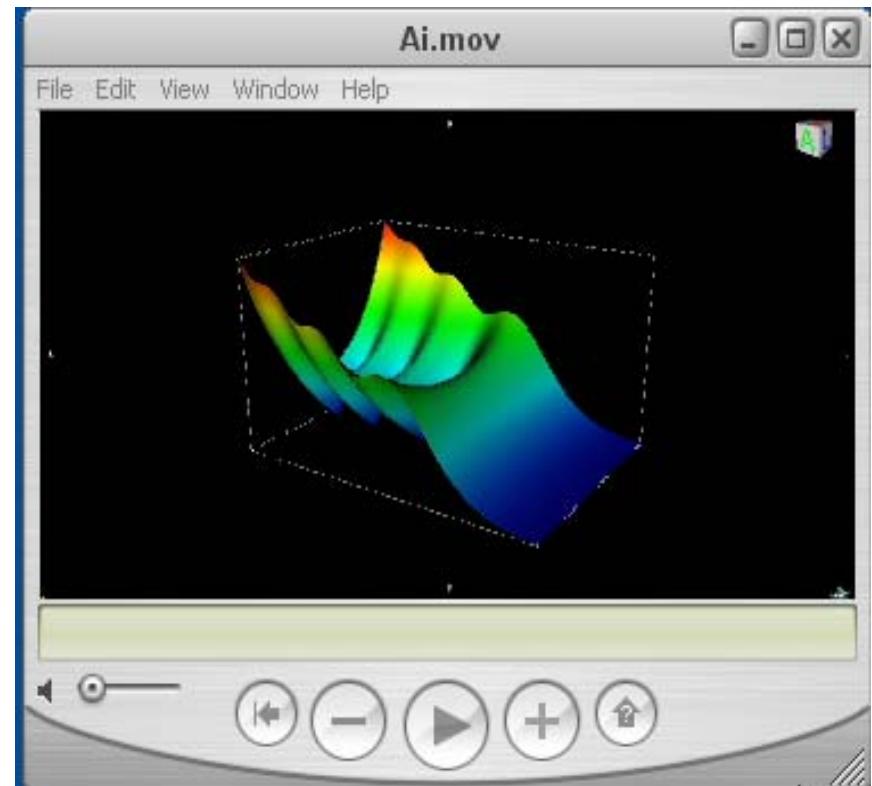


- QuickTime VR is a technology to move the photographic image from the flat 2D world into the definitive immersive experience - complete with 3D imagery and interactive components.
- QTVR movies display three-dimensional places (panoramas) and objects with which the user can interact. With a QTVR panorama, it's as if you are standing in the scene and you can look around you up to 360 degrees in any direction. In a QTVR movie of an object, you can rotate the object in any direction. You also can pan, zoom in or out .



# QTVR for DLMF Figure

- Create QTVR movie using a software which has the function of export QTVR file.
- Open source software OsiriX worked on Apple system has been used to generate QTVR movie for DLMF figures.



# Comparison

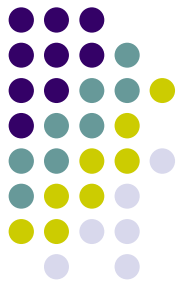


	VRML/X3D	Embed 3D in PDF	Animation	QuickTime VR
User Interaction	3D Navigation using browser, user interaction provided in DLMF VRML	3D Navigation in PDF document	No 3D user interaction	3D Navigation using QuickTime player
Plugin for Web browsers	vrml/x3d browser: Windows: <b>Cortona</b> , <b>BContact</b> , <b>Cosmoplayer</b> . Mac & Linux: <b>FreeWrl</b>	<b>Adobe reader 7.0</b> and later version	Most Web browsers	<b>QuickTime player</b> for most Web browsers
Advantage	Fully functioning of interactive 3D visualization	3D presentation in document;	Good platform portability; No need of special plugins	Good platform portability; No need of special plugins
Disadvantage	Need special browser plugins	partial interactions	No 3D	Pseudo-3d

# Conclusion



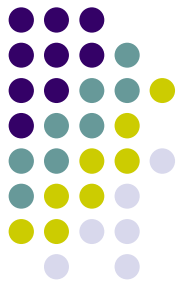
- Web based 3D visualization is an important feature in DLMF, it helps the user to understand complex function.
- The VRML/X3D standard provides a good opportunity to implement web based visualization of DLMF figures. The existing VRML/X3D browsers, such as Cortona, BSContact, Cosmoplayer, and FreeWrl are available for download.
- Various technologies, such as, Embedded 3D in PDF file, animation, Quicktime VR, can help solving the issues of concerning platform portablity and the dissemination of information to as wide as possible of an audience.



# Reference

- Abramowitz, M. and Stegun, I.A. editors 1964. Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables. Vol. 55, National Bureau of Standards Applied Mathematics Series. U.S. Government Printing Office.
- Lozier, D.W. 2002. The NIST Digital Library of Mathematical Function Project. Annals of Mathematics and Artificial Intelligence.
- Wang, A. and Saunders, B. 2005. Web-Based 3D Visualization in a Digital Library of Mathematical Functions. 10<sup>th</sup> International Conference on 3D Web Technology.

# Acknowledgements



- Daniel Lozier, Ron Boisvert, and Bruce Miller for support of the DLMF project