

Fundamental Understanding of Materials Properties Based on the Exact Solution of Many Body Coulombic System by Diffusion Quantum Monte Carlo Method

Yoshiyuki Kawazoe

Institute for Materials Research, Tohoku University,

Sendai 980-8577, Japan

kawazoe@imr.edu

<http://www-lab.imr.edu/~kawazoe/>



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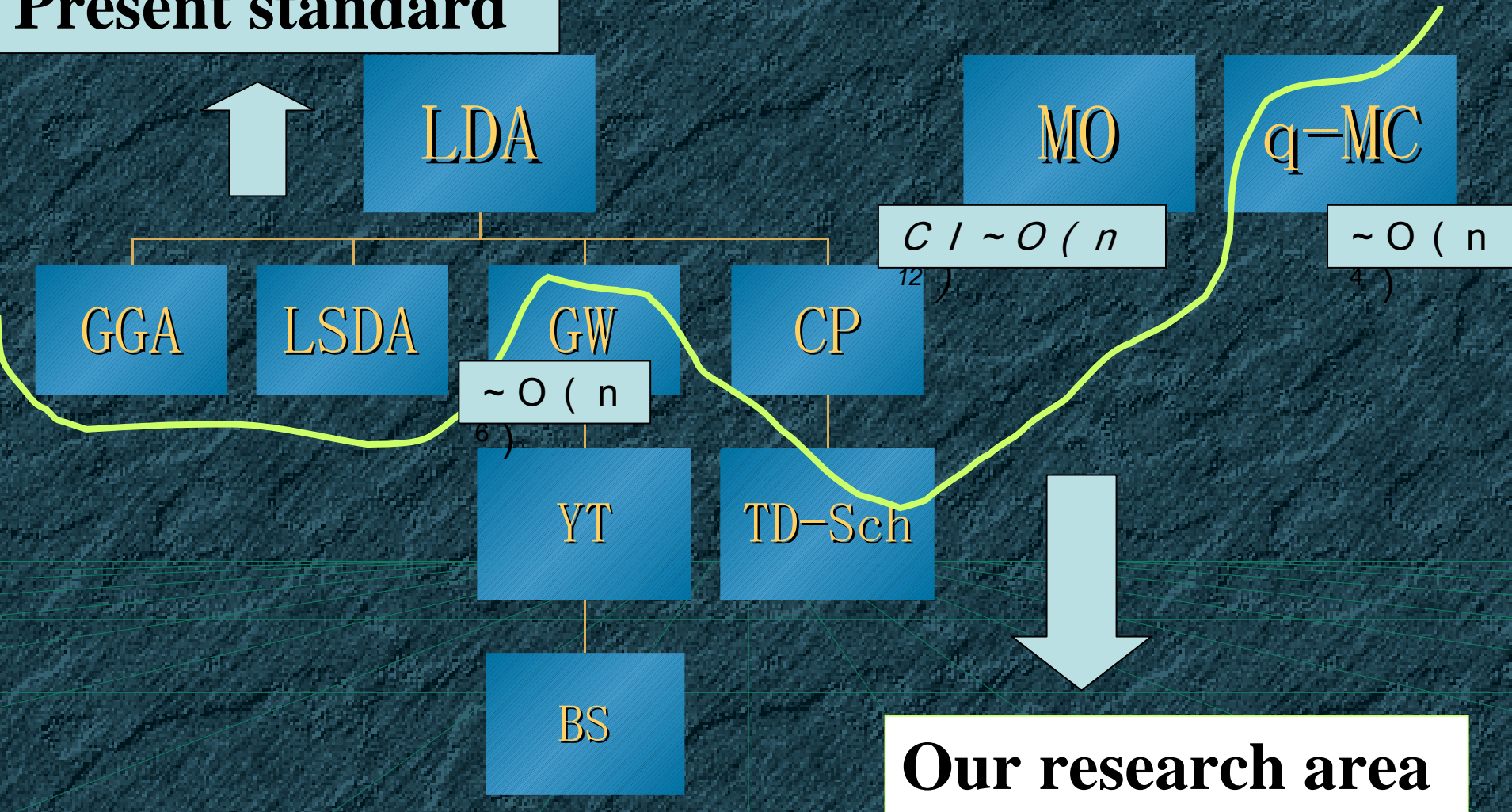
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Why *ab initio* simulation can predict new materials?

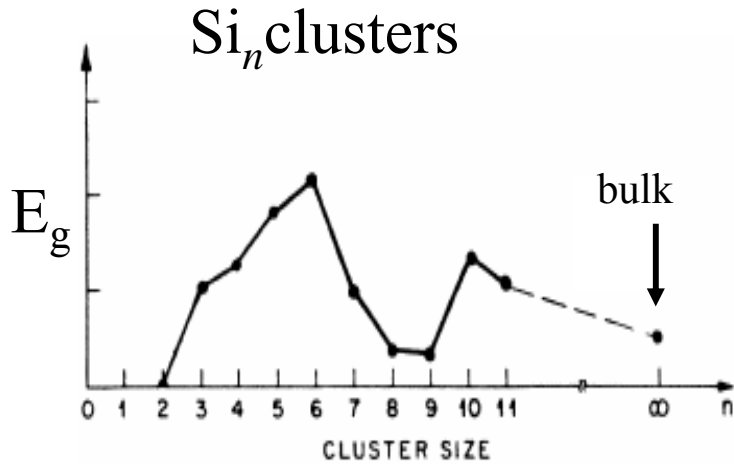
- Objects of the materials research are many body system via Coulomb interaction
 - **Si, steel, DNA all the same !**
- Described by Schroedinger equation (Dirac eq.)
- More than 50 years ago, Dirac already said that “All necessary equations are known, only to solve them!”
- **But!!! Many body problem is too much time consuming... difficult to be solved ... approximations have applied up to the present!**
- **And, many misunderstandings happened!!!**

Ab initio calculation has no experimental parameters but a lot of approximations included!

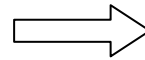
Present standard



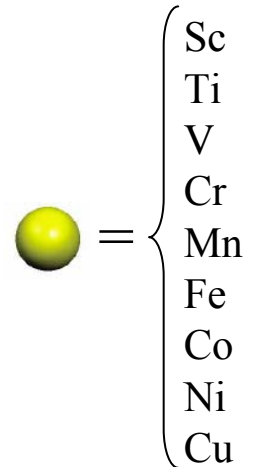
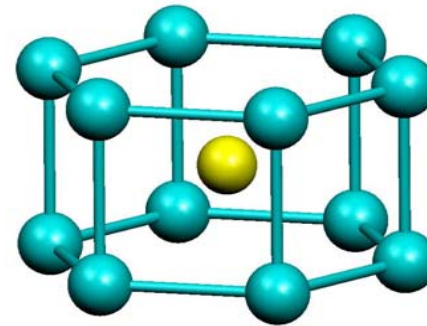
Is LDA good enough?



Size dependence

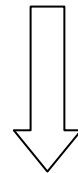


TM@ Si_n clusters



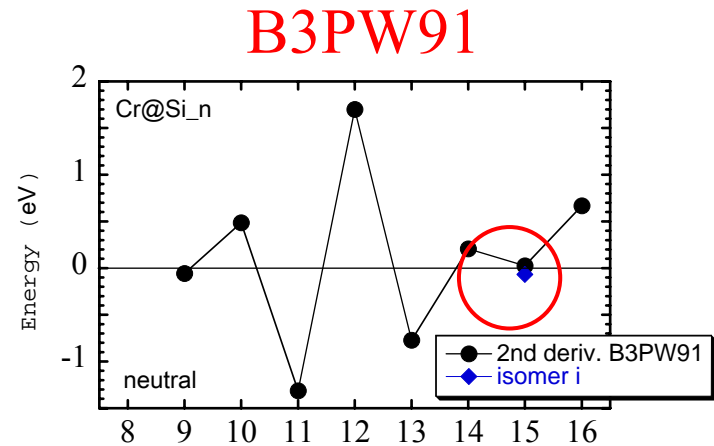
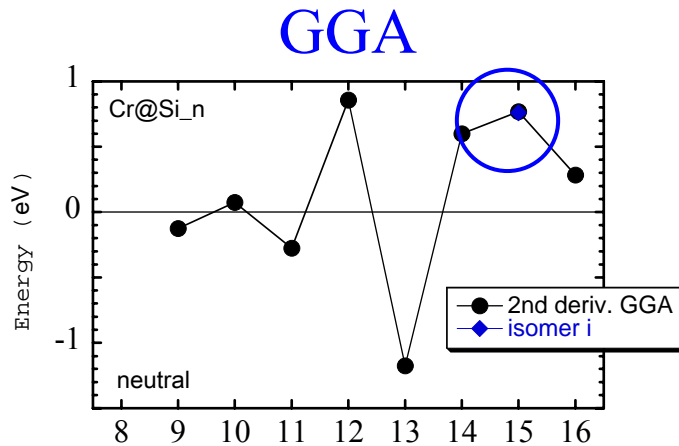
structure

Computer simulation



Normally within LDA → good enough?

Cr@Si_n clusters



※positive energy → stable

- $n=12$ cluster
GGA and B3PW91 → max stable

- $n=15$ (structural isomers)

GGA and B3PW91 give fundamentally different results

→ more accurate method is necessary !

Exact solution of quantum mechanical equation for Coulombic many body system

- Complete solution with electron exchange-correlation → quantum Monte Carlo method
- No restriction for functions
→ diffusion QMC : completely numerical solution ; no restrictions

- Virial theorem : **T and V are not independent!**

General rule for all states in Coulombic system

Coulomb force is the result of geometry ; three dim. space !

Necessary condition for exact theory

$$2T + V = 0 \Rightarrow E = -T = V/2$$

T : kinetic energy of electrons

V : sum of $V_{ee} + V_{eN} + V_{NN}$

- Forming of molecule: nucleus-electron interaction is only attractive condition for molecular stability**

2 independent atoms
H + H

molecule
H₂

energy difference
(molecule-2 atoms)

condition for molecular stability

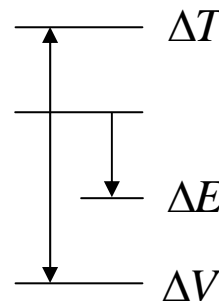
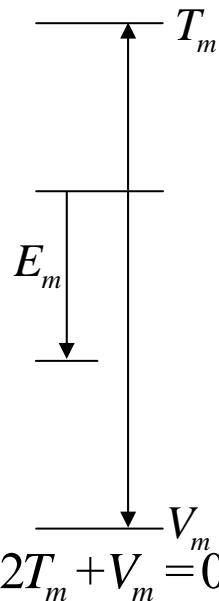
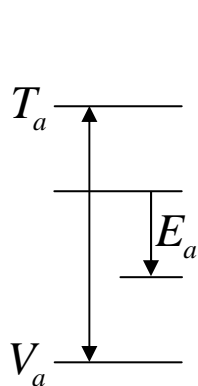
$$\Delta E < 0$$



$$\Delta T > 0$$

$$\Delta V < 0$$

$$-\Delta V / \Delta T = 2$$



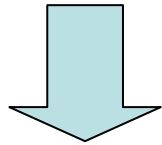
$$2T_a + V_a = 0$$

$$2T_m + V_m = 0$$

$$2\Delta T + \Delta V = 0$$

Origin of molecular stability?

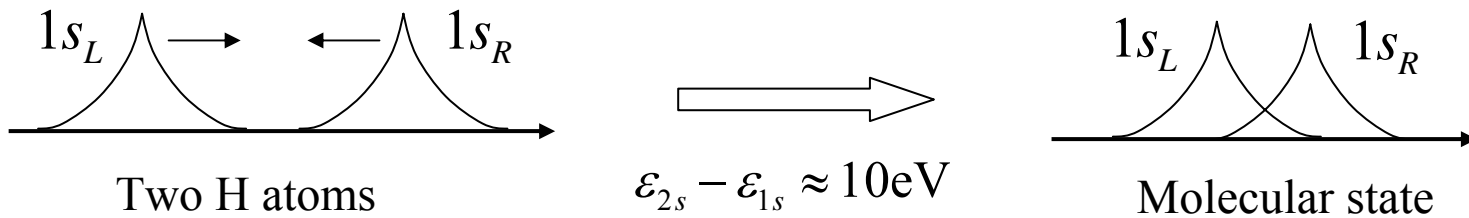
- Electron clouds overlaps?... No!!
- e-e interaction V_{ee} is repulsive!... Not possible!



- Most important interaction to make molecules stable is nucleus-electron V_{Ne} attractive force!

Ex.1 How H₂ molecule formed from 2H atoms?

Two models for H₂ molecular state
HL and MO



Minimum orbitals MO

Heitler-London(HL)model

$$\Psi_{HL}(1,2) \propto 1s_L(1)1s_R(2) + 1s_L(2)1s_R(1)$$

Independent states

$$\Psi_{MO}(1,2) \propto (1s_L(1) + 1s_R(1))(1s_L(2) + 1s_R(2))$$

Molecular state expressed by
1s orbitals only

biding energy	Expl.	HL	MO
$-\Delta E / \text{eV}$	4.74	2.48	2.88

Based on the 1s H atom orbitals only

→ Molecular orbitals are not the linear combination
of atomic orbitals !

Stability condition fulfilled?

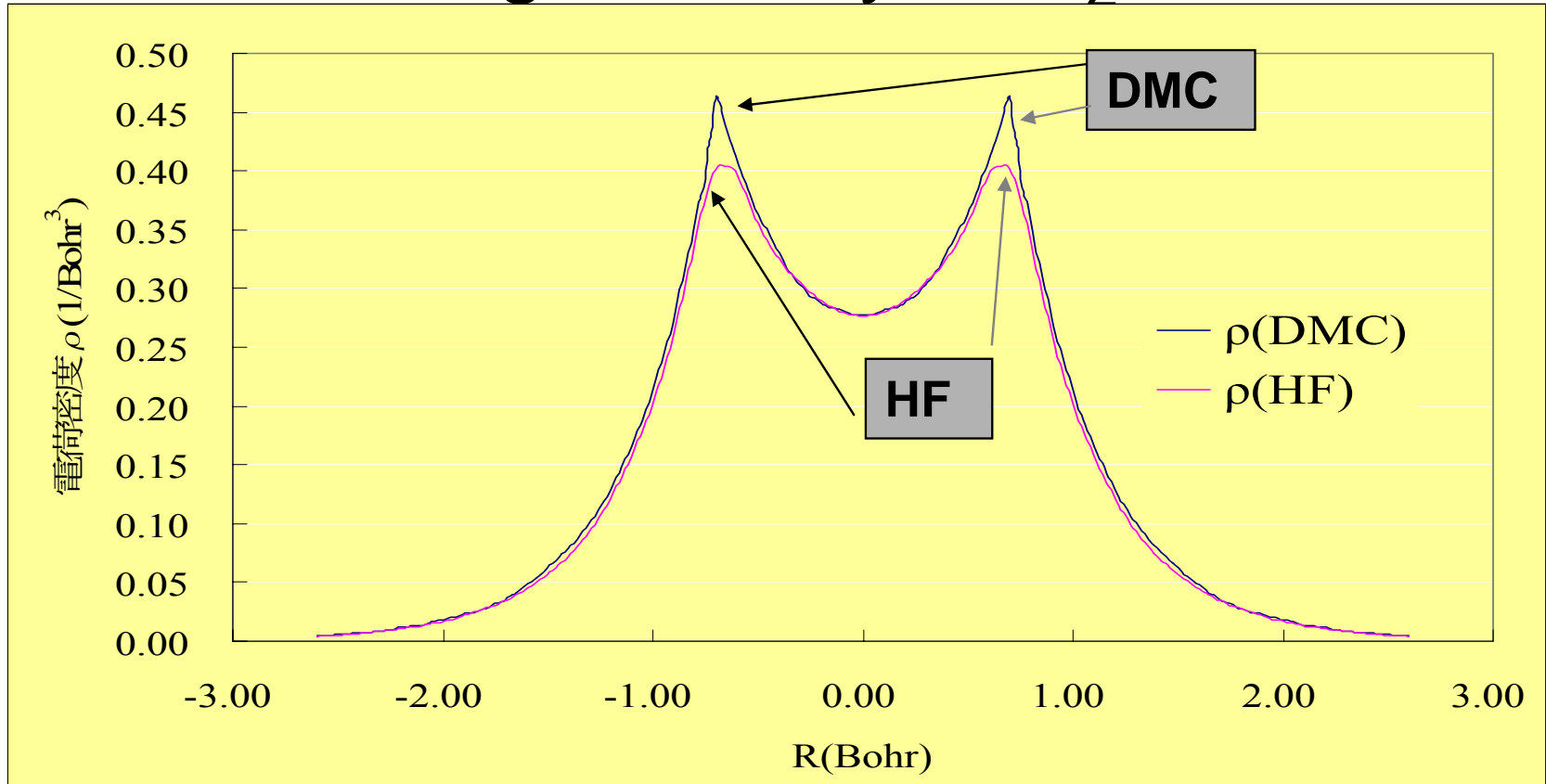
HL and MO give reversed values

(eV) condition	HL	MO	HF	DMC	Expl.
ΔE negative	-2.48	-2.88	-3.63	-4.75	-4.75
ΔT positive	-5.01	-4.22	3.63	4.8(1)	↑ absolutely
ΔV negative	2.53	1.33	-7.27	-9.6(1)	
$-\Delta V / \Delta T$ 2	0.5	0.3	2.0	2.0(1)	

Only by the 1s orbitals, it is not possible to explain the stability of H₂ molecule. Contradict to virial theorem.

Stability of materials should be realized by nucleus-electron int.

Why molecule is stabilized: Charge density in H₂

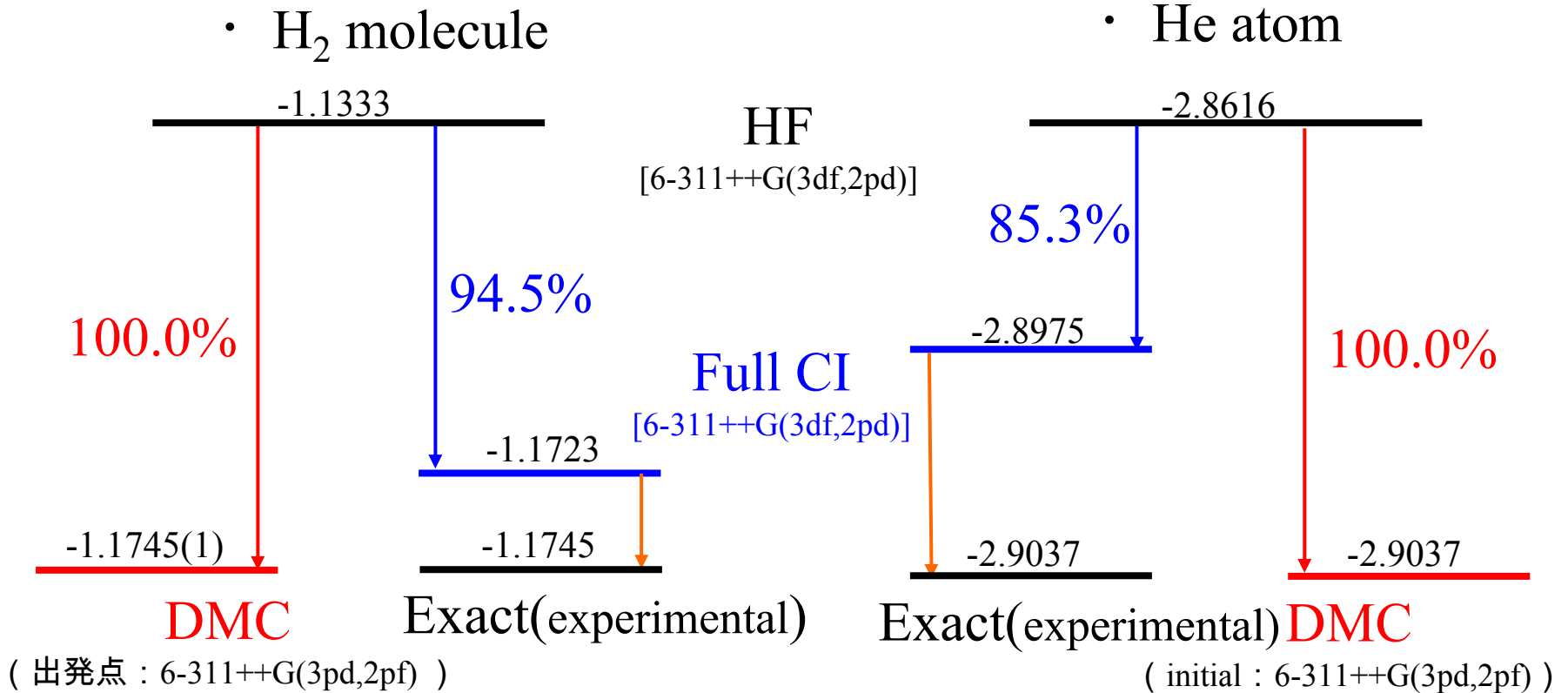


Heitler-London theory (LCAO with 1s atomic orbital) fails!

Virial theorem ($2T+V=0$) should be satisfied!

DMC: kinetic energy increases and potential energy more decreases!

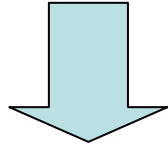
Correlation energy in electron system



Self healing of amplitude by imaginary time evolution
 → Exact numerical solutions

Correct explanation of Hund's multiplicity rule

- Electron exchange interaction?
- At that time, researchers were astonished and tried to explain all of quantum mechanical phenomena by e-exchange.
- Magnetism = Heisenberg model? ... No!!



- Exact numerical calculation including all interactions V_{ee} and V_{eN} !

Diffusion Quantum Monte Carlo (DQMC) method

Imaginary-time evolution projector method

Identical with solving the '*exact*' many-body Schrödinger equation

Electron correlation can be fully taken into account !

Example 1:

New interpretation of *Hund's multiplicity rule* for carbon atom

DMC	E_{total}	V_{ee}	V_{en}	V_{total}	Vrial ratio
Triplet	-37.8280(7)	12.595(6)	-88.253(27)	-75.658(27)	1.99996
Singlet	-37.8107(5)	12.430(4)	-88.053(18)	-75.622(18)	1.99997

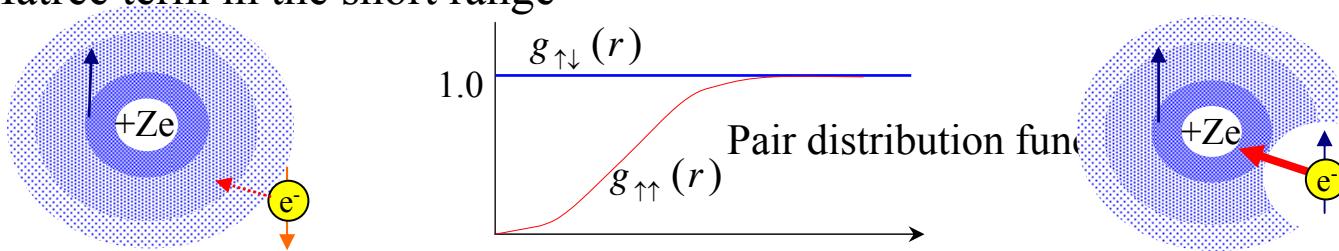
V_{ee} for triplet is larger

V_{en} contributes mostly

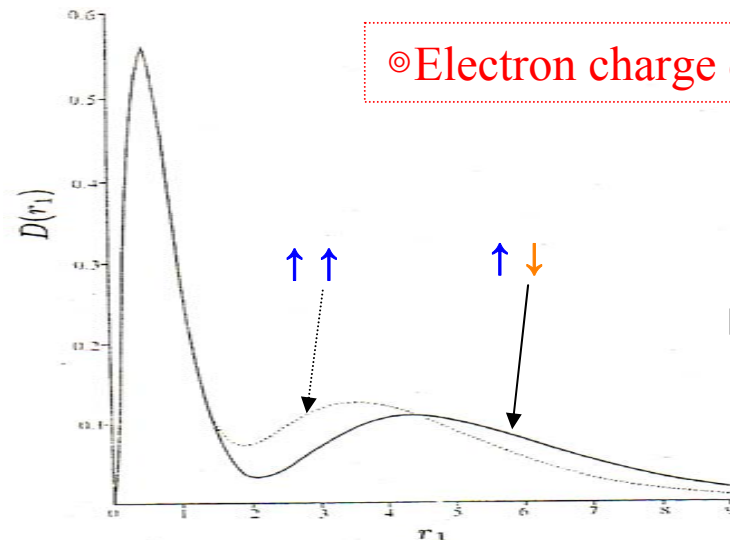
Correct explanation of Hund's multiplicity rule

① Selfconsistent Hartree-Fock Approximation

Exchange term in HF equation weakens the nucleus shielding by Hartree term in the short range



Electronic states having more parallel spins feel more attraction from nucleus



⊙ Electron charge density contracts around nucleus

$$V_{ne}(\uparrow\uparrow) < V_{ne}(\uparrow\downarrow)$$

$$V_{ee}(\uparrow\uparrow) > V_{ee}(\uparrow\downarrow)$$

$$T(\uparrow\uparrow) > T(\uparrow\downarrow)$$

Absolute value estimation of other physical quantities

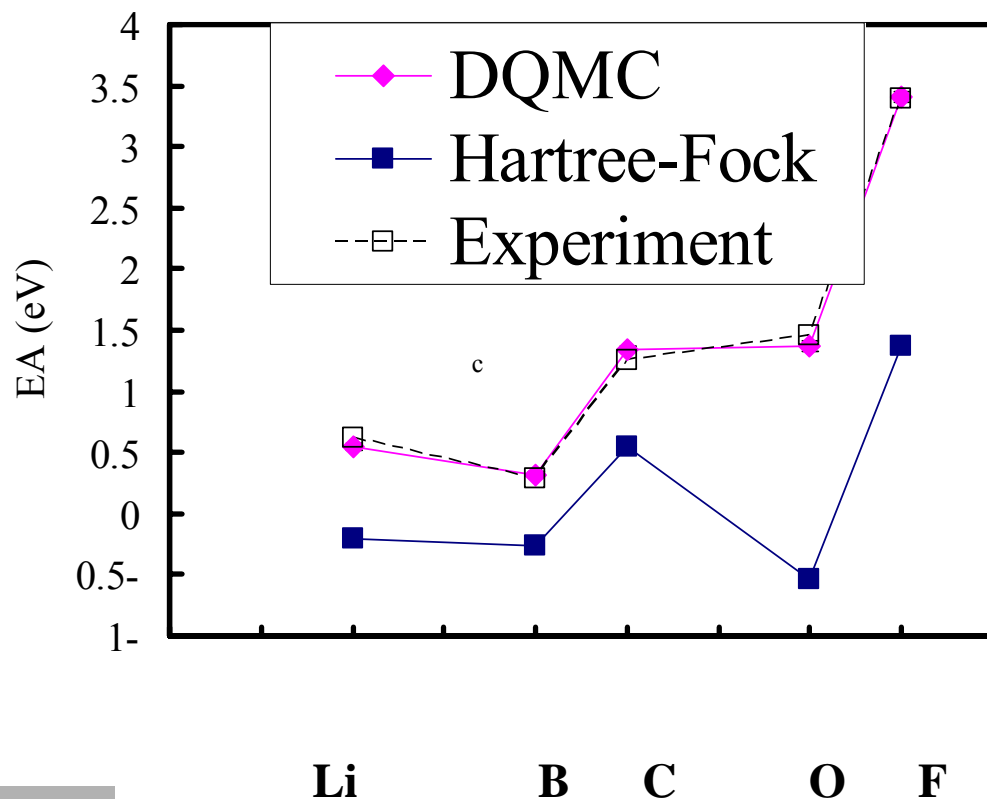
- Electron affinity
- Ionization potential
- HOMO-LUMO gap

- Completed for 3p systems. Computing for transition metals.
- Extension to crystals

Exact Computation of Electron Affinity by Diffusion QMC Method

Electron affinities of light atoms

The electron correlation plays an important role in *EAs* !



computing cost for *ab initio* methods

quantitative Computer cost

DFT

Δ

N^3

→ complete numerical calculation needed!

Quantum chemistry

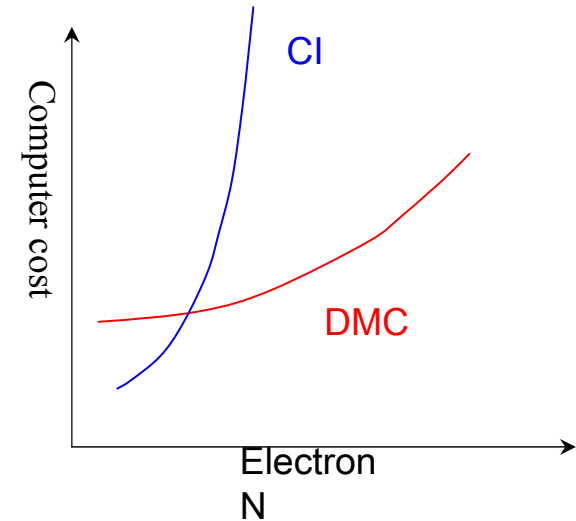
○

$N^6 \sim N!$

Diffusion quantum MC

○

$N^3 \sim N^4$



Ab initio DMS seems to be a better method in future

Collaborators

- V. Kumar, M. Sluiter, J.-Z. Yu, H. Mizuseki, Q. Sun, T. M. Briere, T. Nishimatsu, R. V. Belosludov, A.A. Farajian, J.-T. Wang, Z. Zong, S. Ishii, A. Jain, Q. Wang, G. Zhou, Murgan, C. Majumder, K. Ohno, W. Kohn, S. Louie, H. Yasuhara, B.-L. Gu, P. Jena, Dong, M. Radney, K. Esfarjani, L. Wille, K. Parlinski, S. Tse, S. T. Chui, D.-S. Wang, R.-B. Tao, Z.-Q. Li, Y. Guo, L. Zhou, J. Wu, V. R. Belosludov, Y. C. Bae, A. Taneda, Y. Maruyama, R. Sahara, H.-P. Wang, Z. Tang, T. Ikeshoji, H. Chen, K. Shida, T. Morisato, K. Hongo, H. Kawamura, Khazaei, etc.
- + Experimentalists: M. Kawasaki, T. Oku, T. Hashizume, T. Kondow, S. Tanemura, K. Sumiyama, T. Sakurai, T. Fukuda, etc.
- +Companies: Hitachi, Seiko-Epson, NEC-Tokin, New Japan Steel, Codec, Tore-Dawconing, IBM, etc.